Development and assessment of a modified ζ-method for deflection control of two-way slabs

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Abstract

Deflection control of two-way slabs is a very complex task considering their possible geometries, structural systems, reinforcement layouts and loading histories, as well as the large number of influencing parameters and moment redistribution due to cracking and time-dependent effects of creep and shrinkage. As a consequence, sufficiently simple and accurate methods for deflection control of two-way reinforced concrete slabs have are still missing. In order to fill this gap, a novel method was developed as an adaptation of the ζ-method (as defined by the *fib* Model Code 2010) of interpolating deflections between the uncracked and fully cracked state. The method is based on deflections obtained using linear-elastic analyses in finite element software, which are then transformed through a series of stiffness normalizations to obtain time-dependent deflections due to load and shrinkage. Furthermore, the moment redistribution capacity of two-way slabs is accounted for through adjustments of the interpolation coefficient ζ. The method is tested against available experimental results from literature and compared with results obtained using existing methods such as the crossing beam method. The results show good precision and accuracy of the proposed method and an improvement compared with existing methods.

Keywords: *deflection, two-way slab, serviceability, finite element method, Model Code.*

1. Introduction

Deflections of reinforced concrete (RC) members are an increasingly governing aspect of structural design. Namely, increasing strength of concretes used in construction has not been accompanied by a corresponding increase in stiffness (FIB Bulletin 92, 2019). Coupled with the increasing speed of construction (i.e. earlier loading ages for RC members) and the demand for longer spans, RC structures today are likely several times more flexible than those constructed 50 years ago (FIB Bulletin 52, 2010). All of this is particularly true in the case of RC slabs, members which are most critical in deflection control. Therefore, there is a pressing need for theoretically sound and reliable deflection control methods. However, precisely this phenomenon is among the most complex to model because of the large number of influencing factors (cracking, time-dependent effects, redistribution, among others) and the uncertainties associated with them (Beeby and Narayanan, 2005; Pecić, 2012).

In general, deflection control within the tradition of CEB-FIP and the *fib* Model Code has been based on the ζ-method of interpolating curvatures/deflections between the uncracked and fully cracked state of an RC section/member (CEB, 1985; FIB, 2013). Nonetheless, this approach is not easily applicable

to two-way members such as flat slabs or edge-supported slabs, even though these are the most common types of horizontal structural members in buildings.
So far, deflection control of two-way members has been based on variants of the "crossing beam" or "strip" method in which two-way members are approximated by unidirectional strips of simpler boundary conditions (ACI 435R-95, 2003; Ghali et al., 2002; Gilbert and Ranzi, 2011). Notably, there are two potentially important drawbacks to such methods. First of all, they can contain oversimplifications that ignore the importance of the two-way action of such members and the significant redistribution potential they contain. Secondly, these methods are relatively time consuming. Additionally, existing methods for deflection control of two-way slabs do not take advantage of the tremendous achievements in the field of finite element analysis (FEA) (FIB Task Group 4.4, 2008). Namely, commercial and open-source FEA software has become more and more common in structural design with increasing capabilities. Furthermore, ultimate limit state (ULS) design—of buildings for example—is most often performed on finite element models by carrying out a linear elastic finite element analysis (LFEA) to determine action effects. Such software offers the benefits of modeling arbitrary member geometries with various mechanical properties, as well as modeling arbitrary boundary conditions, load distributions and sequences. Importantly, LFEA can easily yield deformations of such models (i.e. linear elastic deflections).
Therefore, it would be highly beneficial to take advantage of the possibilities offered by LFEA and finite element modeling and develop a deflection control method of two-way members based on linear elastic deflections. Hence, in this paper, a recently developed method (Tošić et al., 2021) is presented along with its comparison with experimental results and the crossing beam method. The method is based on a series of transformations of linear elastic deflections obtained through LFEA that can then be interpolated using a modified ζ distribution coefficient that considers redistribution affects arising from two-way action. The results of the study demonstrate the successful application of the proposed method, its ease-of-use and advantage over the crossing beam method.

2. Modified ζ-method for deflection control of two-way slabs

The basis of deflection control using the ζ-method is the interpolation of deformations between the uncracked and fully cracked state (EN 1992-1-1, 2004; FIB, 2013). In the case of deflections

$$a = \zeta \cdot a_2 + (1 - \zeta) \cdot a_1 \tag{1}$$

where a is the estimated deflection, ζ is the distribution coefficient, a_1 and a_2 are the deflections in the uncracked and fully cracked states, respectively. Equation (1) should be used for deflection control of cracked members, whereas for deflections of uncracked members $\zeta = 0$. The distribution coefficient ζ is determined as

$$\zeta = 1 - \beta \cdot \left(\frac{M_{\text{cr}}}{M}\right)^2 \tag{2}$$

where M_{cr} is the cracking moment, M is the maximum moment along the span of a member (or at its end in the case of cantilevers), and β is an empirical factor equal to 1.0 for first-time loading and 0.5 for repeated or sustained loading. The value of $\beta = 0.5$ also covers the effects of shrinkage and temperature, as well as previous loading and unloading of live loads (CEB Bulletin 235, 1997).
Deflections in states 1 and 2 caused by load, $a_{\text{load},1}$ and $a_{\text{load},2}$, can be expressed as

$$a_{\text{load,i}} = K \cdot \frac{M \cdot L^2}{E_{\text{c,ef}} \cdot I_{\text{i}}^*} \tag{3}$$

where K is a coefficient dependent on the member's boundary conditions and loading (0.104 for a simply supported beam under uniformly distributed load), M is the maximum moment over the element span for the considered load combination, L is the span, $E_{\text{c,ef}}$ is the effective concrete modulus of elasticity, and I_{i}^* is the transformed moment of inertia of a representative cross-section (where M is acting) for state $i = 1, 2$.
Deflections in states 1 and 2 caused by shrinkage, $a_{\text{cs},1}$ and $a_{\text{cs},2}$, can be expressed as

$$a_{\mathrm{cs,i}} = \delta_{\mathrm{cs}} \cdot \varepsilon_{\mathrm{cs}}(t, t_{\mathrm{s}}) \cdot \frac{S_{\mathrm{i}}^{*}}{I_{\mathrm{i}}^{*}} \cdot \frac{L^{2}}{8} \tag{4}$$

where δ_{cs} is a coefficient dependent on the boundary conditions and reinforcement layout (1.0 for a simply supported beam), $\varepsilon_{cs}(t,t_s)$ is the shrinkage strain at time t for a concrete cured until t_s, and S_i^* is the transformed first moment of area of the reinforcement in state i (=1 or 2) relative to the transformed section's centre of gravity.

When considering LFEA, an elastic modulus E_c is input as a material property of the model with each section typically having gross properties, such as the gross concrete moment of inertia I_c. Then, the elastic deflection obtained by LFEA, a_{LFEA}, due to load can be expressed as

$$a_{\mathrm{LFEA}} = K \cdot \frac{M \cdot L^{2}}{E_{\mathrm{c}} \cdot I_{\mathrm{c}}} \tag{5}$$

Considering Equations (3) and (5), the following transformation can be made to express $a_{load,i}$ in terms of a_{LFEA}:

$$a_{\mathrm{i}} = a_{\mathrm{LFEA}} \cdot \frac{E_{\mathrm{c}} \cdot I_{\mathrm{c}}}{E_{\mathrm{c,ef}} \cdot I_{\mathrm{i}}^{*}} \tag{6}$$

As for shrinkage, an assumption is necessary in order to relate deflection due to shrinkage to deflection due to load. Namely, it is assumed that in the representative cross-section, the ratio between curvatures due to shrinkage and due to load is translatable to a ratio between deflections due to shrinkage and due to load:

$$a_{\mathrm{cs,i}} = a_{\mathrm{load,i}} \cdot \left(\frac{1}{r}\right)_{\mathrm{cs,i}} \Big/ \left(\frac{1}{r}\right)_{\mathrm{load,i}} = a_{\mathrm{load,i}} \cdot \left(\varepsilon_{\mathrm{cs}}(t, t_{\mathrm{s}}) \cdot \frac{S_{\mathrm{i}}^{*}}{I_{\mathrm{i}}^{*}}\right) \Big/ \left(\frac{M}{E_{\mathrm{c,ef}} \cdot I_{\mathrm{i}}^{*}}\right) \tag{7}$$

Substituting Equation (6) for $a_{load,i}$ in Equation (7), the following is obtained:

$$a_{\mathrm{cs,i}} = a_{\mathrm{LFEA}} \cdot \frac{E_{\mathrm{c}} \cdot S_{\mathrm{i}}^{*}}{M} \cdot \varepsilon_{\mathrm{cs}}(t, t_{\mathrm{s}}) \cdot \frac{I_{\mathrm{c}}}{I_{i}^{*}} \tag{8}$$

In this way, all deflection components are expressed in terms of the linear elastic deflection obtained using LFEA.

It should be noted that LFEA typically yields as a result linear elastic displacement from which deflections need to be determined, i.e. displacements contain (a) a kinematic component due to the displacement of end "strips" of each considered "strip", and (b) a deflection component produced by curvature along the considered "strip." As an illustration, Figure 1 displays the displacements of a flat slab. The total displacement of the corner panel centre point is 3.33 mm. However, considering a midspan strip in the x direction, the displacement is zero at the slab perimeter but 2.85 mm at the end of the panel span. Therefore, the value a_{LFEA} for the panel centre is a_{LFEA} = 3.33 – 0.5 × (0 + 2.85) = 1.43 mm.

The method proceeds by selecting a unidirectional strip of unit width for which deflections and internal forces are obtained. The geometric properties of the selected strip (such as the x direction strip in Figure 1), can be calculated considering a cross-section of unit width (b = 1 m). When calculating geometric properties, the "average reinforcement ratio" for tensile and compression reinforcement (ρ_m and ρ_m', respectively) should be used (CEB, 1985; Marí et al., 2010), Figure 2.

$$\rho_{\mathrm{m}} = \frac{\rho_{\mathrm{A}} l_{\mathrm{A}} + \rho_{\mathrm{C}} l_{\mathrm{C}} + \rho_{\mathrm{B}} l_{\mathrm{B}}}{l} \tag{9}$$

$$\rho_{\mathrm{m}}' = \frac{\rho_{\mathrm{A}}' l_{\mathrm{A}} + \rho_{\mathrm{C}}' l_{\mathrm{C}} + \rho_{\mathrm{B}}' l_{\mathrm{B}}}{l} \tag{10}$$

where ρ_A and ρ_A' are the reinforcement ratios of tensile and compressive reinforcement at support A; ρ_B and ρ_B' are the reinforcement ratios of tensile and compressive reinforcement at support B; ρ_C and ρ_C' are the reinforcement ratios of tensile and compressive reinforcement at midspan; l_A and l_B are lengths of those parts subjected to negative moments; and l_C is the length of the part subjected to positive moments.

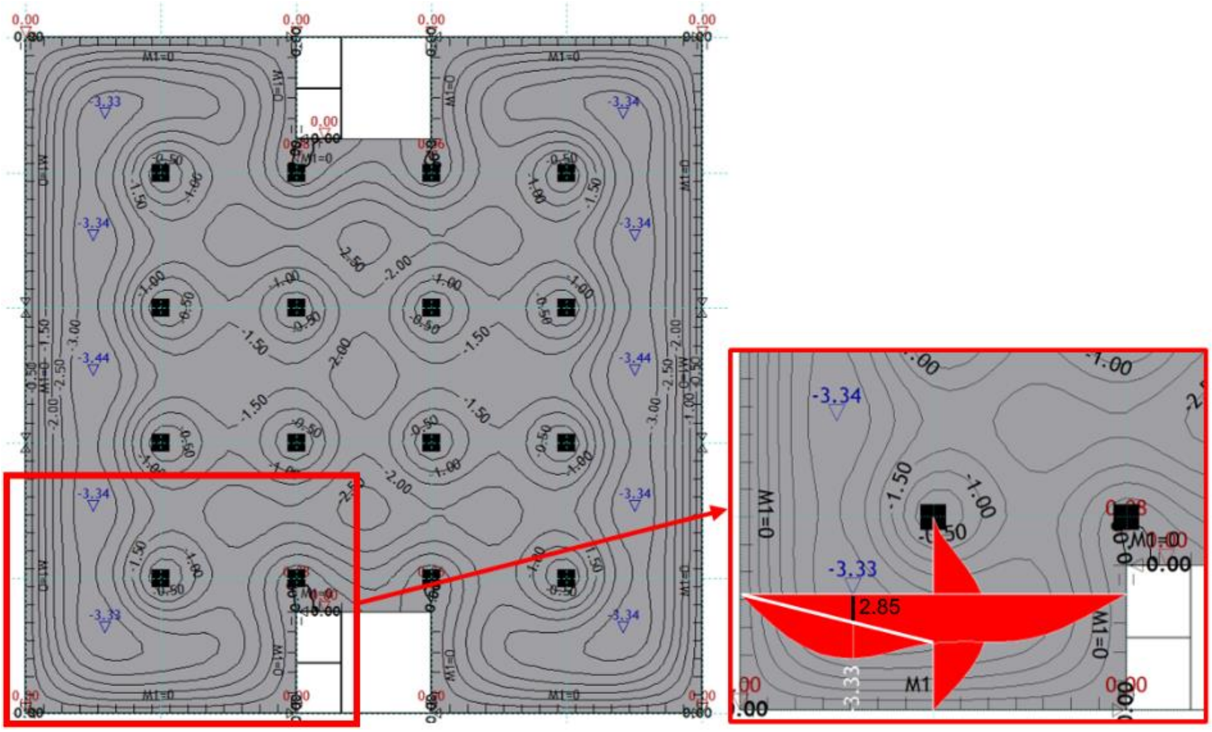

Figure 1. LFEA displacements for a flat slab.

After calculating deflections in states 1 and 2 ($a_i = a_{load,i} + a_{cs,i}$), the deflections a_1 and a_2 need to be interpolated using a distribution coefficient ζ, in case that the cracking moment M_{cr} is exceeded. Due to the greater redistribution capacity of two-way members compared with one-way members and the larger effect of uncracked portions of the slab on deflections, the current formulation of ζ given by Equation (2) was considered inadequate. Therefore, the following is proposed:

$$\zeta = 1 - \beta \cdot \frac{M_{cr}}{M} \tag{11}$$

By reducing the exponent from 2 to 1, the tension stiffening effect is increased. Furthermore, every representative cross-section of a two-way slab has moments in two orthogonal directions, i.e. M_x and M_y. This means that there are also two distribution coefficients, ζ_x and ζ_y, obtained by replacing M in Equation (11) with M_x and M_y, respectively.

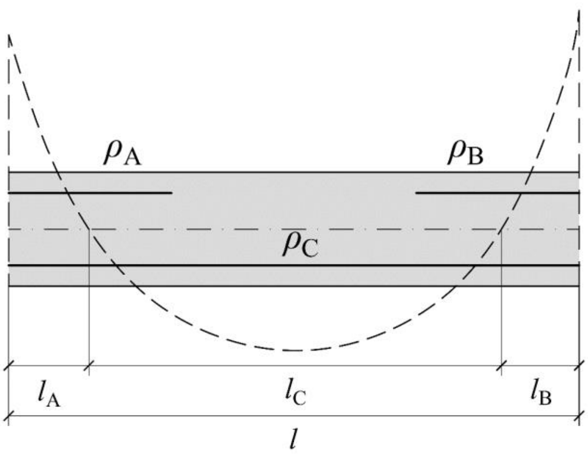

Figure 2. Bending moment diagram defining l_A, l_B and l_C (Marí et al., 2010).

Existing crossing beam methods suggest calculating deflections twice—in the *x* and *y* directions—and adopting an average value of deflections as the estimate (Gilbert and Ranzi, 2011). However, herein, it is proposed to calculate deflections only once (choosing between the *x* and *y* directions) and interpolating using an average distribution coefficient ζ_{mod}:

$$\zeta_{\text{mod}} = \frac{\zeta_x + \zeta_y}{2} \geq 0.75 \cdot \max\{\zeta_x, \zeta_y\} \tag{12}$$

Additionally, a lower bound on the interpolation between ζ_x and ζ_y is imposed in order to ensure that the proposed method has a relatively smooth transition towards one-way slabs as the ratio between spans l_x and l_y increases.
Finally, the deflection is determined as

$$a = (1 - \zeta_{\text{mod}}) \cdot a_1 + \zeta_{\text{mod}} \cdot a_2 \tag{13}$$

3. Long-term experiments on two-way slabs

Another reason for the slower advance in developing two-way member deflection control methods is the scarcity of long-term experiments against which such methods can be tested. Namely, there are only a few studies reported in sufficient detail to allow such comparisons (Gilbert and Guo, 2005; Jaccoud and Favre, 1982; Radnić and Matešan, 2008; Tellenbach and Favre, 1985).
Herein, the studies by Jaccoud and Favre (1982), Tellenbach and Favre (1985) and Radnić and Matešan (2008) will be discussed. The study by Gilbert and Guo (2005) is presented in more detail in (Tošić et al., 2021). The choice of experiments in this study was made on the basis of allowing easy and simple comparison with the crossing beam method predictions.
Among four series of long-term tests, Jaccoud and Favre (1982) tested two series of two-way slabs: series B (consisting of slabs B1, B2, and B3) and series D (consisting of slabs D1, D2, and D3). Slabs from series D are reported in detail by Tellenbach and Favre (1985).
B series slabs were rectangular slabs with a thickness of 120 mm and 4-m spans, continuously edge-supported, Figure 3. Slabs B1 and B2 were reinforced with Ø10 mm bars spaced at 150 mm in both directions, whereas slab B3 was reinforced with Ø12/150 mm and Ø8/150 mm in orthogonal directions. Slab B1 was loaded with a load of 10.85 kN/m^2, and slabs B2 and B3 with 18.76 kN/m^2. The slabs were loaded at the age of 14 days and deflections were measured over a one-year period and reported for the centre point of the slabs. Compressive strength, tensile strength, modulus of elasticity, creep coefficient, and shrinkage strain values were reported as well.

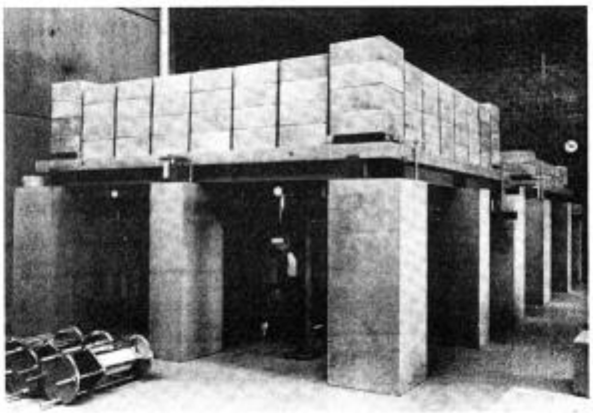

Figure 3. Photo of slab B1 (Jaccoud and Favre, 1982).

Slabs from series D were also rectangular with a 110-mm thickness and spans of 2.7 m. Slabs D1 and D2 were continuously edge-supported, whereas slab D3 was point-supported in the corners, Figure 4. Slabs D1 and D3 had a reinforcement consisting of Ø10/150 mm in both directions. Slab D2 was excluded from the analysis as it had the same reinforcement, but it was placed diagonally relative to the edges. Slabs D1 and D3 were loaded at an age of 21 days with 26.6 and 8.9 kN/m2, respectively, and deflections were measured during a two-year period and reported for the centre point of the slabs. Compressive strength, tensile strength, modulus of elasticity, creep coefficient, and shrinkage strain values were reported as well by Tellenbach and Favre (1985).

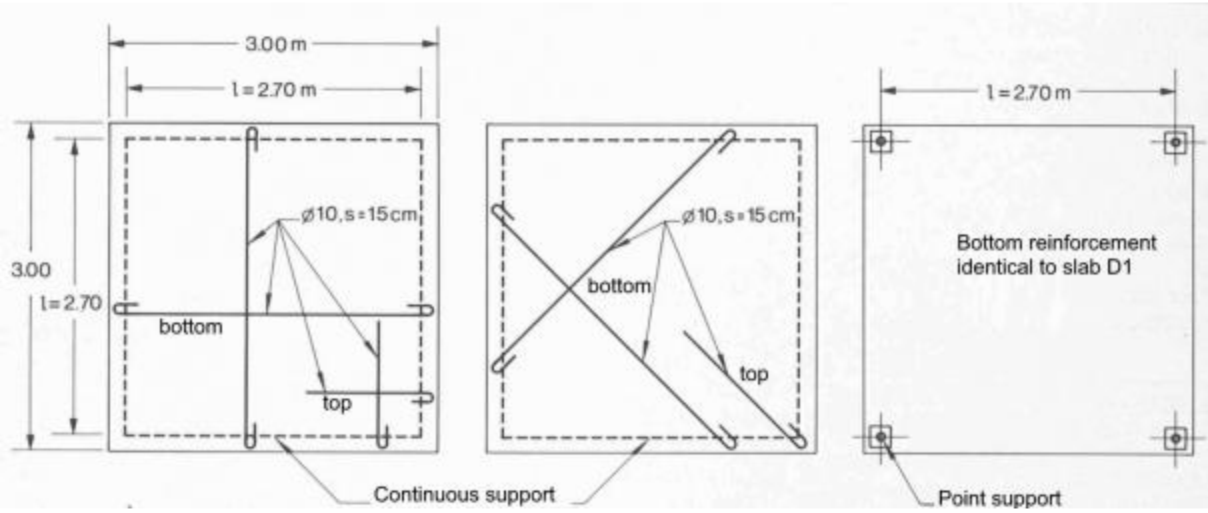

Figure 4. Reinforcement of slabs D1, D2 and D3 (Tellenbach and Favre, 1985)r.

Radnić and Matešan (2008) tested a corner-supported slab with dimension of 1 × 1 m and a thickness of 30 mm, Figure 5. The slab was made of a concrete with a 55 MPa 28-day compressive strength. All mechanical properties, whereas shrinkage and creep were reported in another study (Radnić and Matešan, 2010). The slab was loaded at the age of 92 days with a load of 10 kN distributed over a central area of 0.9 × 0.9 m. Deflections were measured in three points over a one-year period: MD1, MD2, and MD3, corresponding to the slab centre and *x* and *y* edges, respectively.

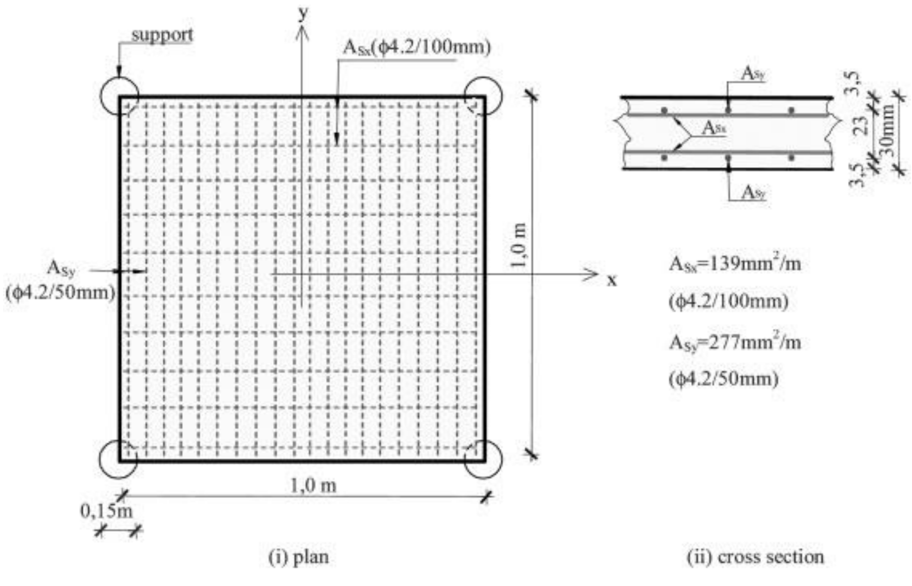

Figure 5. Reinforcement plan of slab MD (Radnić and Matešan, 2008).

4. Comparison with experiments and crossing beam method

In order to assess the performance of the proposed method, deflections for the above-presented experimental tests were calculated using the proposed method ("modified ζ-method") and the crossing beam method. For the modified ζ-method, LFEA was performed using the Tower 8 software (Radimpex, Serbia). The slabs were modelled using the default thin-plate elements available in the software. Mesh size was selected as 2*h*, where *h* is the slab depth. Supports were modelled as point supports and load was applied as uniformly distributed. The only mechanical property necessary for modelling was the modulus of elasticity, E_c, which was entered as the experimentally measured value at age of loading as reported in the corresponding studies. Following an LFEA, bending moments and elastic displacements were obtained for the slabs. Deflections at the centres of the slabs were considered from both directions (*x* and *y*) to assess the sensitiveness of the methods to the choice of strip direction. For calculations using both methods all experimentally reported values were used (tensile strength, modulus of elasticity, shrinkage strain and creep coefficient).

In case of edge-supported slabs, it was necessary to consider only one strip, as shown for the *x*-direction strip of slab D1 in Figure 6a). For corner-supported slabs, such as D3, to calculate the deflection at the panel centre, it was necessary to consider strips in both directions, e.g. the mid-panel strip in the *x*-direction and the column strip in the *y*-direction, Figures 6b) and 6c), respectively.

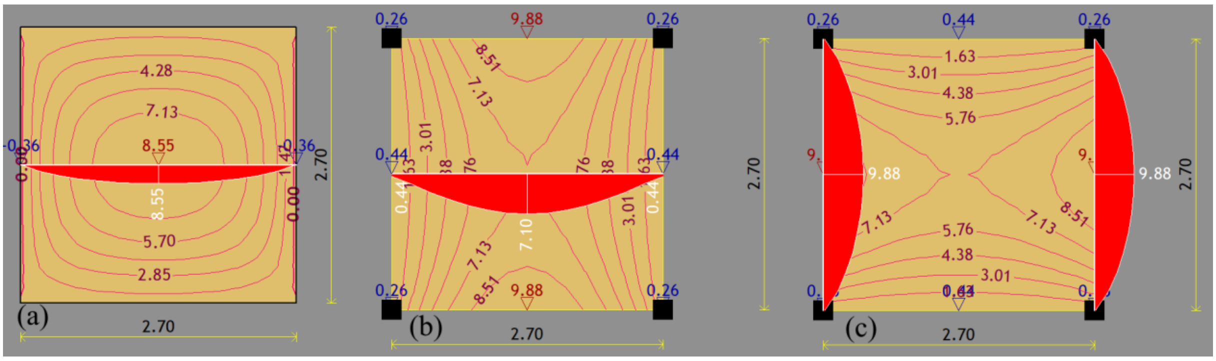

Figure 6. Example of (a) *x*-direction strip for slab D1, (b) mid-panel *x*-direction strip for slab D3 and (c) column *y*-direction strip for slab D3.

In the case of the crossing beam method, the strips were approximated with simply supported beams in both directions. In the case of edge-supported slab, as the spans of the slabs were equal and boundary conditions were symmetrical, half of the load was considered to be transferred in each direction. In the case of corner-supported slab, the full load was considered to be transferred both through mid-panel and column strips (Gilbert and Ranzi, 2011).

Since the reported experiments lasted relatively shortly, the measurements can be considered biased towards earlier measurement times. This is not realistic in practice were deflections are typically determined for a service life of 50 years. Specifically, the coefficient β in Equations (2) and (11) changes from 1.0 to 0.5 immediately after loading, causing a discontinuity when the evolution of deflections over time is modelled. Previously, it has been proposed that β can be formulated as function of time decreasing from 1.0 to 0.5 (Tošić, 2018). Therefore, for the purposes of this analysis, a linear decrease of β from 1.0 to 0.5 over 100 days was adopted, remaining constant at 0.5 after 100 days.

The results are assessed in terms of the calculated-to-experimental deflection ratio, a_{calc}/a_{exp}, which is presented for both methods in Figures 7–9, where "-x" and "-y" refers to the direction of the selected strip. The results presented in the figures demonstrate a very good agreement between experimental results and predictions by both methods, especially when considering the number of influencing parameters and associated uncertainties. Nonetheless, a better performance of the modified ζ-method can be seen.

The a_{calc}/a_{exp} evolves very steadily over time for both methods, although it is more stable for the modified ζ-method and with a smaller scatter of results. Additionally, the results according to the modified ζ-method are closer to experimental values, as the crossing beam method also presents certain significant overpredictions. Therefore, considering the benefits involved with using LFEA—such as the ability to use the same method for ULS verification and deflection control, model arbitrary geometries and loads and perform very quick deflection calculations—the proposed modified ζ-method can be considered to be bringing significant benefits to deflection control of two-way RC members.

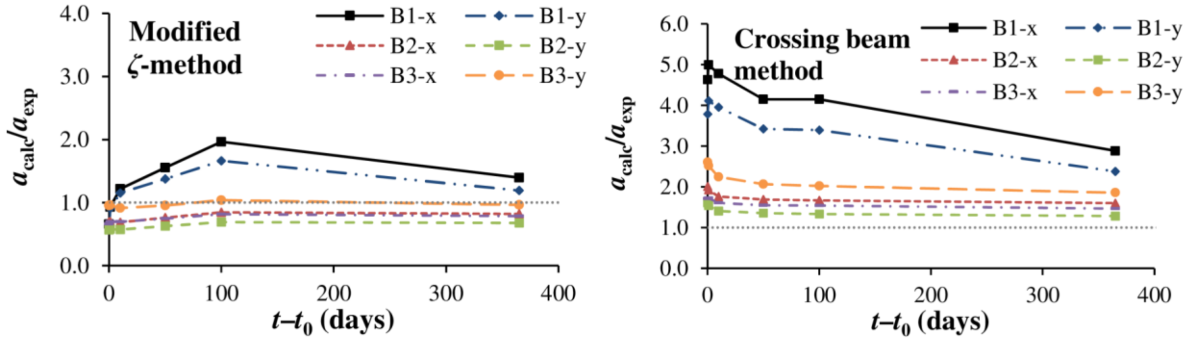

Figure 7. Comparison of the a_{calc}/a_{exp} ratio for the modified ζ-method and crossing beam method and the slabs from (Jaccoud and Favre, 1982).

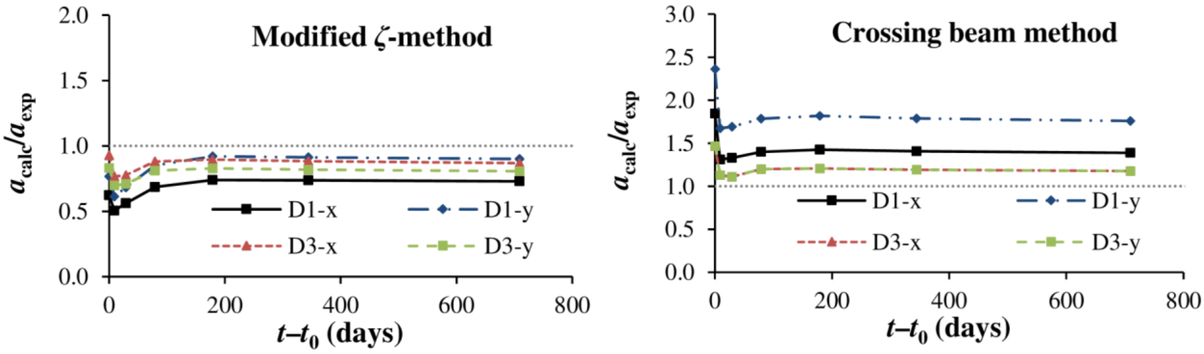

Figure 8. Comparison of the a_{calc}/a_{exp} ratio for the modified ζ-method and crossing beam method and the slabs from (Tellenbach and Favre, 1985).

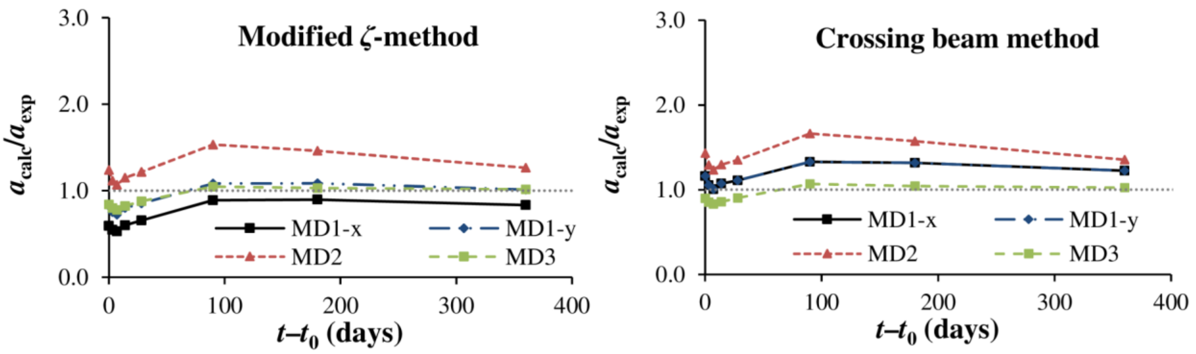

Figure 9. Comparison of the a_{calc}/a_{exp} ratio for the modified ζ-method and crossing beam method and the slab from (Radnić and Matešan, 2008).

5. Conclusions

This paper presents a modified ζ-method for deflection control of two-way RC members, based on linear elastic analysis in finite element software. The method is based on linear elastic deflections and a series of transformations that yield deflections in the cracked and fully cracked states due to load and shrinkage. Modifications to the ζ distribution coefficient consider the elevated redistribution and tension stiffening capacity of two-way members. Based on the proposed method and comparisons with experimental results and predictions using the crossing beam method, the following conclusions are drawn:

- Elastic deflections obtained through LFEA performed in finite element software can form the basis of a deflection control method for two-way RC members.

- Time-dependent deflections caused by load and shrinkage can be obtained by transformations of linear elastic deflections and based on properties of a representative cross-section of a two-way slab.
- The proposed method yields very good, stable and uniform predictions relative to available experimental results; the predictions are slightly better than those based on the crossing beam method with accompanying savings in calculation time and additional versatility of the method.

Through further refinements and calibrations against nonlinear numerical parametric studies, the proposed method can become an accurate and precise, easy-to-use deflection control method that could be incorporated into future revisions of design codes such as the *fib* Model Code.

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