

A Discussion on the Reliability of prEN1992-1-1:2021 Shear Strength Provisions for Fibre Reinforced Concrete Members Without Shear Reinforcement

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Abstract. The Eurocode 2 for the design of concrete structures (EN1992-1-1:2004) is undergoing a revision that will lead to the publication of the second generation of this code to be used across all CEN member countries. Therefore, the impact of the code will reach hundreds of millions of people. Importantly, the Eurocode 2-revision will incorporate many significant changes, amongst which is the introduction of design provisions for fibre reinforced concrete (FRC). In this regard, one of the most important aspects for ultimate limit state (ULS) design will be the consideration of the shear strength of FRC members without shear reinforcement. Such a failure mode is associated with a number of uncertainties, as well as with a potentially-brittle failure response. However, so far, the design models proposed in the Eurocode 2-revision have not been accompanied by a robust reliability-based calibration of the FRC partial factor γ_{SF} . Within this context, this paper presents the results of an investigation on the safety format and the partial factor γ_{SF} for FRC members without shear reinforcement currently provisioned in the draft for the new Eurocode 2 (prEN1992-1-1:2021). Firstly, a database of experimental results on FRC beams is used to determine the model error. On that basis, a probabilistic analysis is performed using the First Order Reliability Method (FORM) to determine adequate values of γ_{SF} for varying target reliability indices. The results of the study show how γ_{SF} values need to be updated in order to reach reliability indices typically considered for ULS design.

Keywords: Fibre Reinforced Concrete, Reliability, Partial Factor, Beam, Database, Eurocode 2.

1 Introduction

Fibre reinforced concrete (FRC) is a structural material that is increasingly acknowledged as an efficient and potentially more sustainable alternative to conventional rein-

forced concrete (RC) [1–3]. A clear advantage of FRC—from the perspective of reducing construction time and material use—is the reduction of steel reinforcement in RC members, and particularly that of shear reinforcement. However, in order for designers to be able to use FRC in such a way, reliable ultimate limit state (ULS) design models are required [4]. This is instrumental, since shear resistance models, especially for members without shear reinforcement, are associated with a large number of uncertainties and shear failures of such members tend to be brittle with potentially catastrophic consequences. Therefore, it has been a topic attracting research interest for decades [5], producing a wide variety of empirical, semi-empirical and mechanical models.

In Europe, the European Committee for Standardization (CEN) has the mandate of producing structural design codes, i.e. the Eurocodes. For RC structures, the current Eurocode 2 [6] (EN-1992-1-1:2004, EC2 in the following) dates back to 2004 (with corrigenda in 2008 and 2010). Since then, numerous advances have occurred, both in the material and in the structural fields, leading to the preparation of a new generation of Eurocodes approximately 10 years ago. Within this new generation, the Eurocode 2-revision [7] (EC2-rev in the following, whose current draft is prEN1992-1-1:2021) will contain a wide range of provisions for new types of elements, concrete types and reinforcement. In particular, an informative annex is planned for steel fibre reinforced concrete (SFRC), with potential expansion in the future covering macro-synthetic fibre reinforced concretes (MSFRC) as well.

Since the CEN Enquiry phase is approaching, this study was conceived with the aim of providing a reliability-based assessment of the EC2-rev shear resistance model for FRC members without shear reinforcement. Also, it is aimed at calibrating the FRC partial factor γ_{SF} required for achieving the required code-prescribed probabilities of failure P_f according to different consequence classes. To achieve this goal, first, the model error was calculated using a database of experimental results. Then, a parametric study was carried out using the First Order Reliability Method (FORM) considering different probability distributions and parameters of input variables. Based on the results, the partial factor γ_{SF} was calibrated based on the target reliability index and failure probability.

2 Description of the Design Model and Assessment of the Model Error

2.1 prEN1992-1-1:2021 Model for the Shear Strength of FRC Members Without Shear Reinforcement

The prEN1992-1-1:2021 provisions in its annex L that, for SFRC members not requiring design shear reinforcement and with longitudinal bars in the tensile zone, the design value of the shear strength should be taken as:

$$\tau_{Rd,cF} = \eta \cdot \frac{0,6}{\gamma_C} \cdot \left(100 \cdot \rho_l \cdot f_{ck} \cdot \frac{d_{dg}}{d} \right)^{\frac{1}{3}} + f_{Ftud} \geq \eta \cdot \frac{11}{\gamma_V} \cdot \sqrt{\frac{f_{ck}}{f_{yd}} \cdot \frac{d_{dg}}{d}} + f_{Ftud} \quad (1)$$

$$\eta = \max \left\{ \frac{1}{1 + 0,43 \cdot f_{\text{Ftuk}}^{2,85}}; 0,4 \right\} \quad (2)$$

where:

- γ_C partial factor for plain concrete (1.50 for persistent and transient design situations)
- ρ_l longitudinal reinforcement ratio
- f_{ck} characteristic concrete cylinder compressive strength
- d_{dg} size parameter describing the crack and failure zone roughness
- d cross-section effective depth
- f_{Ftud} design ultimate residual tensile strength of SFRC (detailed below)
- γ_V partial factor for shear and punching resistance without shear reinforcement (1.40 for persistent and transient design situations)
- f_{yd} design yield strength of reinforcement
- f_{Ftuk} characteristic ultimate residual tensile strength of SFRC (detailed below)

The size parameter d_{dg} is defined as

$$d_{dg} = \begin{cases} 16 \text{ mm} + D_{\text{lower}} \leq 40 \text{ mm} & \text{for } f_{ck} \leq 60 \text{ MPa} \\ 16 \text{ mm} + D_{\text{lower}} \left(\frac{60}{f_{ck}} \right)^4 \leq 40 \text{ mm} & \text{for } f_{ck} > 60 \text{ MP} \end{cases} \quad (3)$$

where D_{lower} is the smallest value of the upper sieve size D in an aggregate for the coarsest fraction of aggregates in the concrete permitted by the specification of concrete EN 206 [8]. It should be noted here that this definition of D_{lower} is adequate for the design stage when the precise aggregate distribution is not known. However, from a physical standpoint, it seems more appropriate to use the maximum aggregate size D_{max} (when known) in Eq. (3).

As for the FRC residual strengths, the characteristic ultimate residual strength is determined as

$$f_{\text{Ftuk}} = \kappa_O \cdot 0.37 \cdot f_{R,3k} \quad (4)$$

$$f_{\text{Ftud}} = f_{\text{Ftuk}} / \gamma_{\text{SF}} \quad (5)$$

where:

- κ_O factor accounting for fibre orientation (taken as 0.5)
- $f_{R,3k}$ characteristic residual tensile strength corresponding to a crack mouth opening displacement (CMOD) of 2.5 mm in the notched beam three-point bending test according to EN 14651 [9]
- γ_{SF} partial factor for FRC (1.50 for persistent and transient design situations)

In fact, Eq. (1) is an expansion of the prEN1992-1-1:2021 formulation for RC members:

$$\tau_{Rd,c} = \frac{0,66}{\gamma_V} \cdot \left(100 \cdot \rho_l \cdot f_{ck} \cdot \frac{d_{dg}}{d} \right)^{\frac{1}{3}} \geq \frac{11}{\gamma_V} \cdot \sqrt{\frac{f_{ck}}{f_{yd}} \cdot \frac{d_{dg}}{d}} \quad (6)$$

A difference can be observed between the left hand-sides of Eqs. (1) and (6), in the ratios $0.6/\gamma_C$ and $0.66/\gamma_V$. From Eqs. (1) and (6) it can be seen that the FRC contribution

to the shear resistance mechanism is expressed through a positive effect of the design residual resistance f_{Rtd} and a negative effect of the factor η that accounts for higher crack widths that can be achieved when using FRC (penalizing the shear-transfer actions of concrete).

2.2 Experimental Database for Model Error Assessment

In order to perform a reliability-based calibration of the FRC partial factor, the model error δ of Eq. (1) should first be determined. The model error is defined as the ratio between actual behaviour/shear strength measured in experiments and shear strength predicted by models (this is a safe assumption as it neglects the geometrical and material deviations in the actual tests). For the present case, this will be performed for a model equation identical to that for design purposes, but where partial factors taken as 1.0 and considering average values for the material properties in the model equation.

Therefore, for this step, a large and representative database of experimental results is required. For this purpose, the database collected by Lantsoght, freely available online [10,11] was considered. This database comprises 488 results on SFRC beams with longitudinal reinforcement and without shear reinforcement, sourced from 65 individual studies. The range of parameters of the original database is shown in Table 1 under the “Original database” column. All reported results were from simply supported beams tested in three- or four-point bending; the majority had rectangular cross-section, but some presented also flanges (as a first estimate, all types of cross-sections were considered). Importantly, residual strength was not reported in the database, because not all studies reported these values, for a number of reasons. However, a large number of fibre properties is reported, as well as a large variety of steel fibre types [11].

Since the range of parameters was very wide, three filtering criteria were imposed:

- a) Concrete classes between C12 and C120 were considered (mean compressive strengths between 20 and 128 MPa);
- b) Only beams with a longitudinal reinforcement ratio smaller than 4% were considered;
- c) Only beams with a clear shear span-to-effective depth ratio larger than 2.0 were considered.

These criteria were selected so as to limit the cases to the range of typical structural concrete compressive strengths, reinforcement ratios, as well as to eliminate cases where a direct load transfer to the support was occurring due to a short shear span-to-effective depth ratio. In total, using the three criteria, the number of specimens was reduced to 332. Approximately 90% of the results were on beams with $f_{\text{cm}} < 70$ MPa, effective depth between 100 and 500 mm and a longitudinal reinforcement ratio between 1.0% and 3.5%. The fibre volume fraction of 329 out of the 332 beams was below 2.0% (160 kg/m³ for steel fibres) and for 296 beams it was below 1.5% (120 kg/m³ for steel fibres).

Table 1. Range of parameter values in the SFRC database compiled by Lantsoght [10].

Parameter	Original database <i>n</i> = 488		Filtered database <i>n</i> = 332	
	Min	Max	Min	Max
b_w (mm)	50	610	50	610
h (mm)	100	1,220	100	1,220
d (mm)	85	1,118	85	1,118
l_{span} (mm)	204	7,823	459	7,823
a/d (–)	0.46	6.00	2.22	6.00
a_v/d (–)	0.20	5.95	2.00	5.95
ρ_l (%)	0.37%	5.72%	0.37%	3.70%
f_y (MPa)	276	900	276	610
f_{cm} (MPa)	9.8	215.0	20.2	111.5
V_f (%)	0.2%	4.5%	0.2%	4.5%
λ (–)	25	191	25	191
f_{uf} (MPa)	260	4,913	260	4,913

l_{span} – clear span of the beam; a/d – shear span-to-effective depth ratio measured from left side of loading plate to left side of support; a_v/d – clear shear span-to-effective depth ratio measured from face of loading plate to face of support; f_y – yield strength of steel reinforcement; V_f – fibre volume fraction; λ – fibre aspect ratio (ratio of fibre length to diameter); f_{uf} – tensile strength of fibres

2.3 Model Error Calculation

Since the residual tensile strengths for SFRC in the database of experimental results were not reported, they had to be estimated. For this purpose, regressions presented in Figs. 1a (for $f_{R,3}$) and 1b (for $f_{R,1}$) were developed using a statistical analysis of experimental results using the EN 14651 standard test [9], as reported by Venkateshwaran et al. [12], Tiberti et al. [13], Galeote et al. [14], and other experimental programs conducted at the Structures and Materials Technology Laboratory (LATEM) of the Polytechnic University of Catalonia (UPC).

The database includes a large variety of concrete mixes, with a range of compressive strengths of 15–117 MPa, volume fraction of fibres 0.33–2.52%, fibre aspect ratios 35–110, fibre tensile strength 1000 to 3000 MPa and fibre modulus of elasticity (E_f) 190000–210000 MPa. Figure 2 shows a good fit between the proposed linear regressions and the observed data.

The model error δ was estimated as $\delta = V_{\text{experimental}}/V_{\text{model}}$. The model shear strength V_{model} was calculated as $\tau_{\text{model}} \cdot b_w \cdot z$, where τ_{model} is the shear stress calculated using Eq. (1), b_w is the width of the cross-section and z is the internal lever arm that is taken as $0.9 \cdot d$. All partial factors were taken as 1.0, reported experimentally measured average values were used and residual strengths predicted by the regressions shown in Fig. 1.

Based on the 332 results, the mean model error was found to be 1.461, with a standard deviation of 0.394 and a coefficient of variation (CoV) of 26.9%.

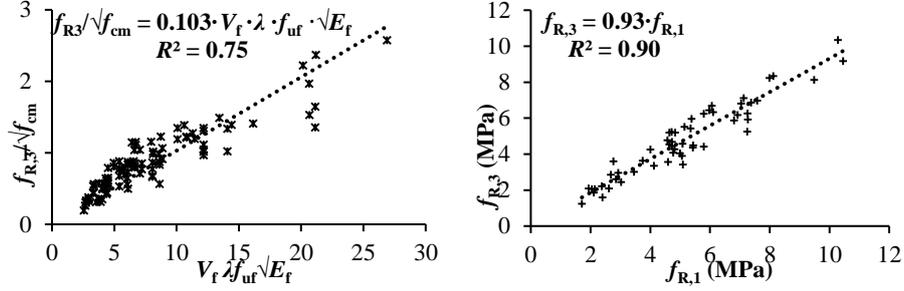


Fig. 1. Correlations used to assess $f_{R,3}$ (left) and $f_{R,1}$ (right) of the SFRC.

3 Calibration of the FRC Partial Factor γ_{SF}

3.1 Design Set and Reliability-Based Analysis

To assess the reliability of the prEN1992-1-1:2021, a set of design cases was defined, Table 2, following the methodology by Bairán and Casas [15]. A range of thicknesses typical for buildings and bridge-deck slabs, beams, footings, and mat foundations was selected, corresponding to the range of 200–1000 mm. The cross-sectional width was considered constant and equal to 300 mm, since the shear strength is linearly dependent on it. The effective depth was determined as $d = h - d_s = h - 50$ mm. The variables in Table 2 produce 420 combinations of geometry, longitudinal reinforcement, concrete class and aggregate size (expressed via d_{dg}). Different values of d_{dg} were considered in order to investigate the effect of aggregate size. The values considered for d_{dg} were the minimum and maximum values permitted (16 and 40 mm, respectively), as well as an intermediate value in order to observe the dependency on d_{dg} .

The process consisted of generating a design set of load shear stresses (Fig. 2) between 0.53 and 2.72 MPa using the currently proposed values of the resistance factors $\gamma_c = \gamma_{SF} = 1.50$ and $\gamma_v = 1.40$. FRC residual flexural capacities were limited to $f_{R,3k,min}$ and $f_{R,3k,max}$ of 3 and 10 MPa, respectively. The range was further divided in quarters, so that five design loads were obtained for each case. Considering the 5 design loads, a total of 2100 design cases (420×5) were generated.

Table 2. Range of variables in the design set.

Parameter	Values of parameters in the design set						
b (mm)	300						
h (mm)	200	400	600	800	1000		
ρ_l (-)	0.002	0.005	0.010	0.015	0.020	0.025	0.030
f_{ck} (MPa)	30	50	70	90			
d_{dg} (mm)	16	24	40				

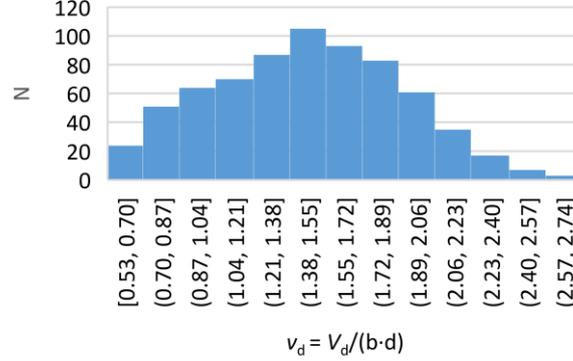


Fig. 2. Histogram of design load shear stresses.

The reliability of each design case was assessed using the reliability index β , related to the probability of failure (P_f) by $\beta = -\Phi^{-1}(P_f)$, where Φ is the cumulative standard normal distribution. FORM [16,17] was used to estimate β .

A design failure was identified when a zero or negative value was found in the limit state function $G = V_R - V_S = \delta \cdot V_{R,model} - V_S$, where $V_{R,model}$ is the shear resistance predicted by the model and V_S is the shear load. Considering G a function of random variables, the probability of failure was computed as the probability of obtaining a negative value of G :

$$P_f = P(G < 0) = P(\delta \cdot V_R - V_S < 0) \quad (7)$$

To calculate $V_{R,model}$, the prEN1992-1-1:2021 model was used without the safety factors and using the observed average values of the materials and geometry variables. The set of random variables and the corresponding distribution functions used are summarized in Table 3. The model error was selected as lognormally distributed according to the recommendations of the Joint Committee on Structural Safety (JCSS) and taking into account that it is a variable that only takes on positive values [18].

Table 3. Definition and distribution of random variables.

Variable	Description	Statistical model	Mean value (μ)	CoV
δ	Model error	Lognormal	1.461	0.269
f_c	Compression strength	Lognormal	$f_{ck} + 8$ MPa	0.050–0.128
f_{Ftu}	Residual strength at w_u	Lognormal	$1.412 \cdot f_{Ftuk}$	0.2
Δb	Geometrical error in section width	Normal	$0.003 \cdot b \leq 3$ mm	$\frac{4 + 0.006 \cdot b \leq 10}{\mu_b}$ mm
Δd	Geometrical error in effective depth	Normal	10 mm	1

3.2 Calibration of the Partial Factor γ_{SF} for the prEN1992-1-1:2021 Model

To establish the relationship between γ_{SF} and β , the required f_{Ftu} has to be designed for each element of the design set, for different values of γ_{SF} . For this purpose, $V_{Rd} = V_{Sd}$ is imposed and the reliability index is computed for each case. The design shear load V_{Sd} was assumed to be deterministic; therefore, the computed reliability index refers to the probability of reaching a shear strength (V_R) smaller than the design resistance (V_{Rd}): $P(V \leq V_{Rd}) = \Phi(\beta_R)$ where β_R is the resistance reliability index, equal to $\beta_R = \alpha_R \cdot \beta$. α_R is the resistance sensitivity coefficient. For usual conditions this coefficient may be taken as 0.8 [19].

The reliability indexes associated to the prEN1992-1-1:2021 model for the shear strength of FRC members without shear reinforcement (Eq. (1)) were assessed for a range of safety factors γ_{SF} varying between 1.30 and 2.00, Fig. 2.

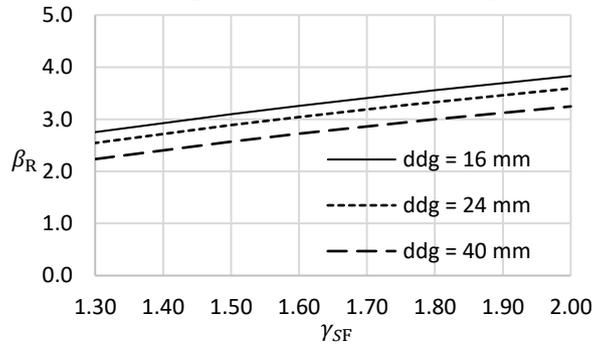


Fig. 2. Variation of the resistance reliability index with respect to γ_{SF} .

As seen from Fig. 2, the reliability index increases with the value of γ_{SF} , and decreases with increasing values of the parameter d_{dg} . The obtained values of β_R for $\gamma_{SF} = 1.50$ are 3.10, 2.89 and 2.57 for d_{dg} equal to 16, 24 and 40 mm, respectively.

As a reference, a target reliability index for ULS verifications for a period of 50 years and medium consequences of failure is $\beta_{target} = 3.8$. Then, the target resistance reliability index $\beta_{R,target} = 0.8 \cdot 3.8 = 3.04$. Therefore, it can be seen that the target reliability index is achieved (and even is slightly safe) when $d_{dg} = 16$ mm, i.e. when D_{lower} is neglected (as in the case of lightweight aggregate concrete structures). At the same time, increasing values of d_{dg} lead to a reduction of the achieved reliability index and, potentially, to a need for the increase in the FRC safety factor γ_{SF} : to achieve exactly $\beta_{R,target} = 3.04$, γ_{SF} should be 1.46, 1.59 and 1.83 for d_{dg} equal to 16, 24 and 40 mm, respectively.

4 Conclusions

This paper presents a reliability-based calibration of the FRC partial factor γ_{SF} for the shear design of FRC members without shear reinforcement according to the EC2-rev (prEN1992-1-1:2021) model. For this purpose, a database of experimental results was used for assessing the model error, after which a reliability analysis was carried out to

calibrate the FRC partial factor. Based on the obtained results, the following conclusions can be drawn:

- Based on a probability analysis and a parametric study of 2100 individual cases, it was determined that the β_R values associated with $\gamma_{SF}=1.50$ are 3.10, 2.89 and 2.57 for d_{dg} equal to 16, 24 and 40 mm, respectively. Hence, in the two cases with d_{dg} larger than 16 mm, the resistance reliability index results less than the code target for medium consequences of failure.
- In order to achieve the target resistance reliability index for ULS verifications for a period of 50 years and medium consequences of failure ($0.8 \cdot 3.8 = 3.04$), γ_{SF} values of 1.46, 1.59 and 1.83 are needed for d_{dg} equal to 16, 24 and 40 mm, respectively. However, for practical purposes, a uniform value of γ_{SF} is desirable. This may be attained through modifications on the safety factor format or on the model, which will be the subject of future research.

The results of this study can serve as a starting point for a more in-depth reliability-based analysis of the shear resistance model for FRC members without shear reinforcement to be provisioned in the future Eurocode 2. Future studies should cover a wider range of parameter values (in particular the fibre type and properties) and contain a deeper analysis of the influence of the d_{dg} parameter, so that recommendations for potential code modifications could be made.

5 Acknowledgements

This study has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 836270. This support is gratefully acknowledged. The authors also wish to express their acknowledgement to the Ministry of Economy, Industry and Competitiveness of Spain for the financial support received under the scope of the projects PID2019-108978RB-C32. Any opinions, findings, conclusions, and/or recommendations in the paper are those of the authors and do not necessarily represent the views of the individuals or organizations acknowledged.

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